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A method for evaluating the position deformation distribution of multi-joint robots caused by mechanical bending

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Abstract. This paper introduced a methodology for evaluating the position deformation distribution of multi-joint robots caused by mechanical bending. The concept of multi-joint robot performance distribution was defined and discussed. The multi-joint robot performance distribution can be divided into two levels including workspace expression and performance distribution in the workspace. The relationship between the operating point in the workspace and the joint angle was analyzed. The performances of the operating point was calculated by single configuration and all configuration performance analysis, and the performances of multi-joint robots was expressed based on the workspace expression. The productions of workspace expression and performance distribution were studied by the analysis of the 6R robot's position deformation distribution caused by mechanical bending. Experimental study showed that the method can describe the position deformation distribution of multi-joint robots and it can provide guidance for the application of multi-joint robots.

Key words. Multi-joint robot, position deformation distribution, mechanical bend, configuration.

1. Introduction

Multi-joint robots are considered as typical industrial robots and have been widely used in machining industry [1]–[3]. In practice, there are three characters to affect the position result of the robot:

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1. **Accessibility**. It is an important index to describe the measured range of multi-joint robots and it is determined by the structure parameters of multi-joint robots. However, the position of robot may be influenced by the tension or compression in the edge area of workspace or in the configurations which are difficult to achieve.

2. **Flexibility.** It is generally accepted that the flexibility is proportional to the configuration variation. However, the position of robot may be influenced in the area where the flexibility is low.

3. **Position accuracy**, which is related to a certain configuration of an operation point in the robot's workspace. It may be influenced by many factors, such as the error of manufacturing & assembly, gravity, temperature and inertia force and so on. It may show different characteristics in different configuration and different position. The position of the robots end effectors presents a certain uncertainty as the result of the influence of many factors above[4].

In order to define the position accuracy, VDI/VDE 2617-9[5] proposed a series of concepts, indexes and methods for determining the measurement. Due to the variation of position accuracy, ASME B89[8] defined the precision detection methods for single-point measurement and length measurement by multi-configuration.

Generally, the position accuracy methods of robots can be divided into 3 types: interval analysis method[6], Monte Carlo method[7] and directly measuring method[8]. However, it is still not easy to describe the position accuracy distribution completely.

Vrba[9]studied the relationship between the precision of grating encoder and the position accuracy of 6-joint measurement system. The position accuracy evaluation algorithm was purposed, and the distribution of multi-joint measurement system in the workspace based on the precision of grating encoder was drawn. However, only the hardware parameter is considered in this method; and the comprehensive factors that may further improve the accuracy of 6-joint robot are still missing.

In this paper, the concept of multi-joint robot performance distribution was defined and discussed. The productions of workspace expression and performance distribution were studied by analyzing the 6R robot's position deformation distribution caused by mechanical bending. Experimental study showed that the method can describe the position deformation distribution of multi-joint robots and it can provide guidance for the application of multi-joint robots.

2. Definition of multi-joint performance distribution

Take the 3-joint manipulator (Figure 1) for example, the pose of manipulator can be determined as **formula** (1) when the structure parameters are known.

$$P = A_1 A_2 A_3 \tag{1}$$

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & l_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & l_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, i = 1,2,3 \tag{2}$$

Where θ_i——Joint angle; α_i——Joint twist; l_i——Bar length; d_i——Joint offset.

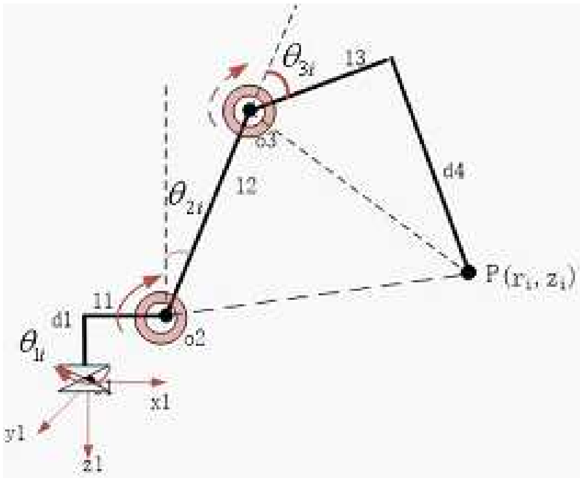

Fig. 1. 3-joint manipulator

As follow as **formula** (1), the end point's position (r_i,z_i) of 3-joint manipulator can be calculated. At the same time, the performance of the manipulator in the pose can be determined.

Therefore, the multi-joint robot performance distribution includes two levels: the workspace expression ($) and the performance distribution (C) based on ($):

$$R = \{\$, C^{(i)}|_{i=1,\ldots,n}\} \tag{3}$$

Where **i** is a class of robot performance and **R** is a class of robot performance distribution.

2.1. Workspace expression

In order to describe the multi-joint robot workspace, we take Figure 1 for example. The workspace of the 3-joint manipulator is shown in Figure 2. Figure 2 (a) shows the half-section of the manipulator workspace. Figure 2 (b) shows the perspective of the manipulator workspace.

As shown as **formula** (2), the workspace of multi-joint robot can be determined by the scope of joint rotation when the bar length, the joint offset and the joint twist are known. Therefore, ($) can be built by the discretization method of rotating the joints.

The end point coordinates (P) of **n**-joint robot can be calculated as **formula** (4).

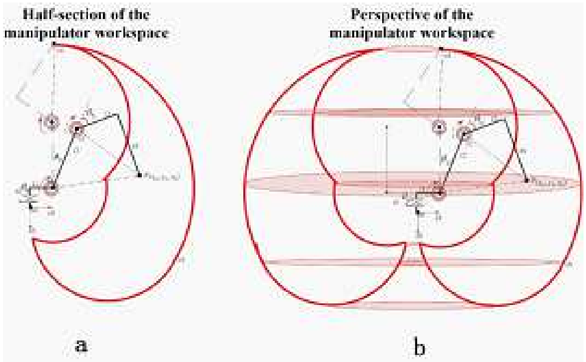

Fig. 2. 3-joint manipulator's workspace

A_i is the matrix of the i^{th} joint.

$$P = \prod_{i=1}^{n} A_i \tag{4}$$

Suppose the maximum rotation angle of **n** joint is θ_{nmax}(i=1...n). θ_{nmax} can be divided into **h** equal parts (θ_{nave}=θ_{nmax}/**h**). ($) of robot can be expressed as **formula** (5):

$$\$ = \{P_j = A_1(\theta_{1a} - k_1\theta_{1s}) * ... * A_i(\theta_{ia} - k_i\theta_{is}) * ... * A_n(\theta_{na} - k_n\theta_{ns})|_{j=1.....n*h,k=1...h}\} \tag{5}$$

Where k_i N & $0 \leq k_i \leq h$. $A_i(\theta_{ia}$-$k_i\theta_{is})$ means the pose matrix of joint **i** when the angle is (θ_{ia}-$k_i\theta_{is}$).

2.2. Multi-joint robot performance distribution

There are many poses in the point of the workspace. The poses are determined by the angles of the joints. They exhibit different performance when the angles of the joints change. The index $C^{(i)}$ can be used to express the performance of robots. The relationship between $C^{(i)}$ and the angles (θ_i(i=1...n)) of robot may be as shown as**formula** (6).

$$C^{(i)} = f^{(i)}(\theta_1, \theta_2, ..., \theta_n) \tag{6}$$

Where $f^{(i)}(\theta_1, \theta_2, ..., \theta_n)$ is the expression for one of the multi-joint robot performances.

Many performances of multi-joint robot, like accuracy, flexibility, rigidity, thermal deformation and so on, can be analyzed by the method above. Then the different performance can be expressed on the workspace expression (Figure 3).

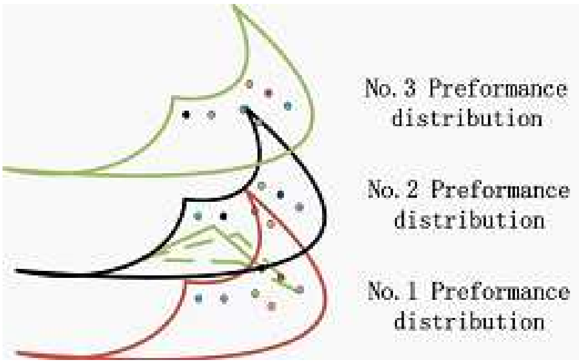

Fig. 3. Multi-robot performance distributions

3. Experiments of position deformation caused by mechanical bending

In order to verify the effectiveness of the performance distribution of multi-joint robots, the GRB3016 6-joint robot (Figure 4) was used in experiments.

Fig. 4. GRB3016 robot

The link mechanism of GRB3016 robot may cause bend deformation due to the weight of mechanical parts (Figure 5). Therefore, the position of robot may be different from the theoretical value. The new position matrix is calculated considering the change of the second joint angle and the position deformation can be obtained. The position deformation distribution by the weight in the workspace can be expressed. Therefore, the position deformation distribution can be described in the cylindrical coordinate plane.

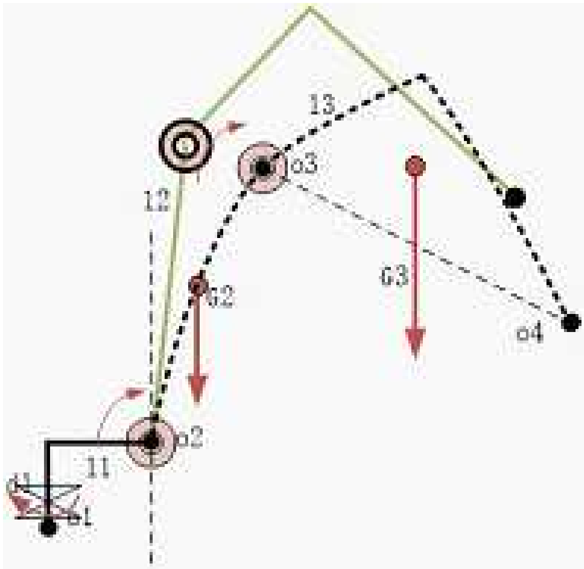

Fig. 5. Bend deformation of GRB3016 robot

3.1. Workspace expression of GRB3016

The DH parameters of GRB3016 are shown as Table 1. The maximum rotation angles of joint 2 and joint 3 are divided into 10 equal parts. The workspace map are drawn by **formula** (4) and shown as Figure 6.

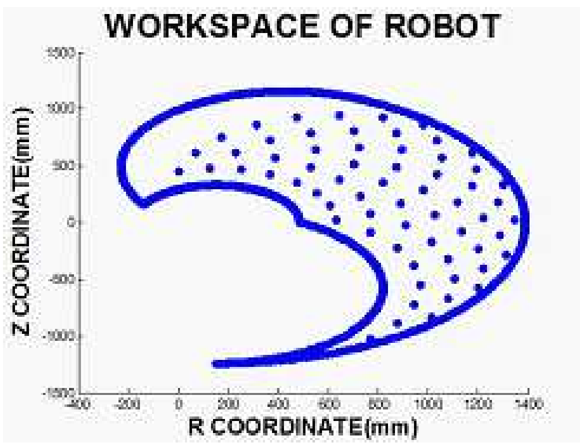

Fig. 6. Workspace of GRB3016 robot

Table 1. DH parameters of GRB3016 robot

	Range of θ_i	l_i(mm)	d_i(mm)	$a_i(\pi)$
A1	[0,2π]	150	0	$\pi/2$
A2	[-$\pi/2$, $\pi/3$]	570	0	π
A3	[arctan|d4/l3|-5π/6, arctan|d4/l3|]	150	0	-$\pi/2$
A4	[0,2π]	0	-650	-$\pi/2$
A5	[0,3π/2]	0	0	$\pi/2$
A6	[0,2π]	0	-105	π

3.2. Deformation performance distribution of GRB3016

As shown in Figure 7, the relationship between the mechanical parts' weight of robot and the torque of joint can be analyzed[10]:

$$\begin{aligned}
&\delta\theta_{2i} = c_2[(G_2\left|\tfrac{l_2}{2}\right| + G_3\left|l_2\right|)\sin\alpha + G_3\frac{2\sqrt{l_3^2+(\frac{d_4}{2})^2}}{3}\cos\beta] \\
&\sin\alpha = \sin(\theta_{2i}+\tfrac{\pi}{2}) \\
&\cos\beta = \cos(\theta_{3i}+\arctan(\left|\tfrac{2l_3}{d_4}\right|)+\theta_{2i}+\tfrac{\pi}{2})
\end{aligned} \tag{7}$$

Where c_2 is the flexibility coefficient, c_2=9.137836×10-61/(N*m), G_2=G_3=100N.

According to **formula** (7), the angle deformation $\delta\theta_{2i}$ of second joint, which is caused by the weight when the angles of the second joint and the third joint are determined, can be calculated together with the robot's parameters and the distance from the gravity center to the joint center.

The end point position deformation distribution can be obtained as shown in Figure 8 according to **formula** (7). The degrees of deformation were represented by different colors. Figure 8 (a) shows the position deformation δd of each working point in the cylindrical coordinate plane. Figure 8 (b) illustrates the distribution of position deformation δd in the workspace.

In order to verify the effectiveness of Figure 8, the position accuracy measurement of GRB3016 was carried out by a standard 3D model. Several points in different region of Figure 8 (a) were selected and the coordinates of selected points can be compared with that in the 3D model. Table 2 showed that coordinate errors of the selected points in different region of Figure 8 (a).

Table 2. Coordinate errors of selected points (mm)

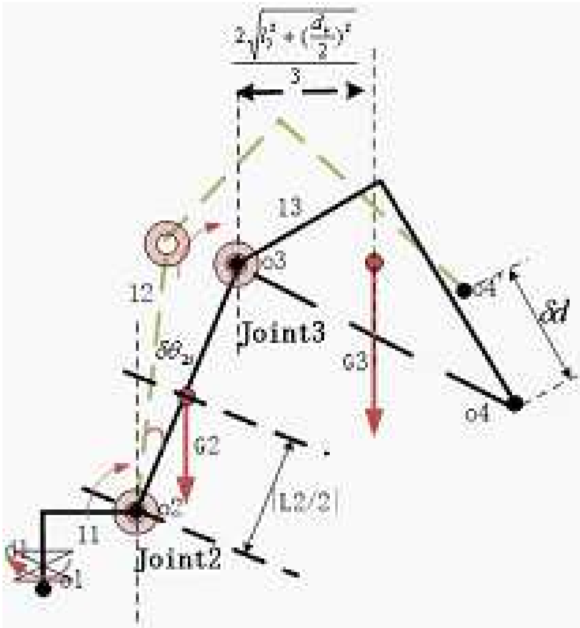

Fig. 7. Robot position deformation model

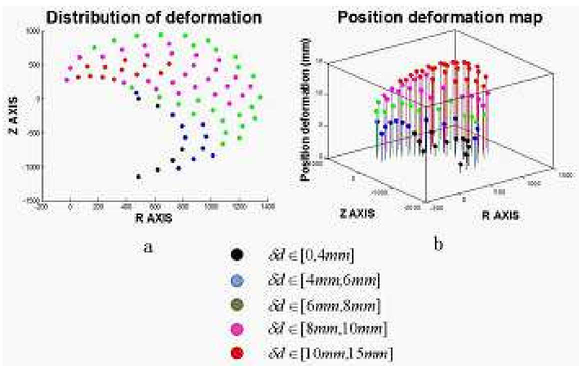

Fig. 8. Robot position deformation distribution

Region	$Point_1$	$Point_2$	$Point_3$	$Point_4$	$Point_5$	$Point_6$
Black	0.23	3.24	2.63	2.75	3.74	3.01
Blue	4.85	5.58	4.13	4.97	5.03	5.93
Green	7.24	7.63	7.26	6.47	6.79	6.27
Pink	8.67	9.27	8.26	9.41	9.53	8.89
Red	10.45	14.76	12.48	14.06	13.49	11.75

As shown in Table 2, the robot position deformation was reasonable and the robot position deformation distribution can be used in the actual operation to describe the deformation performance.

4. Conclusion

A method for evaluating the performance distribution of multi-joint robots was proposed in the paper. The distribution of performance can be divided into the workspace expression and the performance distribution based on the workspace expression. The weight deformation distribution of 6-joint robot was taken in the experiment, and the results proved that the multi-joint robot performance distribution method can express the robot performances digitally, quantitatively, accurately and intuitively. It provides an analysis method and guidance for the robot application.

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